EXPERIMENTAL DETERMINATION OF THE DRAG COEFFICIENT OF A LIQUID DROPLET IN THE PROCESS OF DEFORMATION AND BREAKUP IN A TURBULENT GAS FLOW

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Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 9, No. 1, pp. 152-156, 1968

In practice, velocity calculations for droplets of known size or determination of the energy consumed in the acceleration or breakup of droplets in a turbulent gas flow involves a number of assumptions [1-3]. It is usual to neglect the liquid structure of the droplet, its deformation, its deviation of shape from the spherical, and the pressure in the flow of not one, but many droplets.

These assumptions may be justified in the presence of a high degree of atomization for the liquid, when the fine droplets behave like rigid spherical particles, and thus to employ the drag coefficient C_x for a sphere. However, even when it is known that the droplet is deformed in interaction processes with the gas flow it is necessary in theoretical calculations to use the drag coefficient for a rigid sphere [4, 5].

There is no united opinion about the variation of the drag coefficient for a deforming liquid droplet as it breaks up in a turbulent gas flow [4-6].

Thus, Prandtl [6] considers that the coefficient has a value close to 0.5 and that its subsequent deformation in the flow is accompanied by an increase in that value.

In investigating the breakup of droplets ranging in size from 2.0 to 3.9 mm and from 0.5 to 5.0 mm in a turbulent gas flow Volynskii [4] and Lane [5] took C_x as constant and equal to 0.44, i.e., corresponding to the region of the self-similar regime for flow over a sphere in the range of Reynolds numbers $R = 1 \cdot 10^3 - 3 \cdot 10^5$. Lane, in particular, notes that in the course of breakup there is a transition from the spherical to the lenticular shape with a reduction in the aerodynamic drag coefficient.

However, as our experiments have shown, the assumption of such a value for the drag coefficient of a liquid droplet from 2 to 5 mm in diameter is not justified, since it corresponds to an unreal situation.

We will consider a liquid droplet transported vertically downward by an accelerating gas flow. As the positive direction we take the direction of the velocity vectors of the flow and droplet.

The equilibrium condition is expressed as

$$dF_p = dF_1 + F_2 , \qquad (1)$$

where $dF_r = ma$, $a = dW_k/d\tau$, $dF_1 = dF_c = 1/8 C_x \pi d^2 \rho U^2$, and $F_2 = mg$.

Here, dF_r is the resultant force by which a droplet with mass m acquires an acceleration a on a certain interval, dF_1 is the force exerted on the droplet by the flow, equal in magnitude to the drag, and F_2 is the force of gravity.

We write Eq. (1) in the complete form:

$$m \; dW_k / d\tau = {}^1/_8 C_x \pi d^2 \rho U^2 + mg . \qquad (2)$$

Equation (2) is difficult to solve for C_x owing to the lack of data on the variation of absolute droplet velocity W_k, the area of maximum cross section (projection of the body on a plane perpendicular to the direction of motion), and relative velocity U during the process of acceleration, deformation, and breakup.

Below we present conditions and results of an experimental determination of drag coefficient for water droplets 3.34, 3.0, 2.45-2.66, and 2.0 mm in diameter in a turbulent accelerating air flow.

By means of high-speed motion-picture photography on an interval 10 mm long we recorded the state of droplets of the above-mentioned size. The breakup process takes a very short time (0.01 sec) and, for a droplet of a particular size, occurs at a perfectly definite "critical" gas velocity.

To obtain a smoother picture of droplet deformation before breakup, the camera was focused on the convergent part of a plexiglas Venturi tube of rectangular cross section. To obtain a large volume of information about droplet deformation we took a series of photographs along the length of the convergent part of the Venturi tube at each investigated gas velocity in the throat (10, 15, 20, 25, 30, and 35 m/sec).

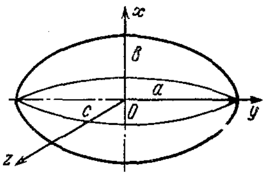

Fig. 1

Thus, a particular, identical, degree of deformation for droplets of given size developed on different intervals of the convergent zone, and the greater the gas velocity in the throat, the sooner the given degree of deformation was attained.

In analyzing the experimental data we assumed that the degree of deformation for droplets of given size is a function of relative velocity only.

In [7] it was established from pictures of a deforming droplet taken in mutually perpendicular planes with two synchronized high-speed motion-picrure cameras that the planar dimensions of the droplets differ only slightly from a circle. We assumed, therefore, that the theoretical shape of the deformed droplet was a flat ellipsoid of revolution $b < a = c$ about the x-axis (Fig. 1).

Actually, in the initial stage of deformation, called the "pulsating" stage in [7], the front of the droplet has a somewhat greater radius of curvature than the tail. Unstable equilibrium between dynamic pressure acting on the droplet and surface tension leads to a constant oscillation of the droplet surface about the shape of a rotational ellipsoid of revolution. In the second stage of "regular" deformation of the droplet, when correspondence between the actual and theoretical shapes is already complete, further flattening takes place with a gradual reduction of the semiaxis ratio $k = b/a$.

The volume of a rotational ellipsoid of revolution about the x-axis $V = 4/3\pi abc$. Considering that $a \approx c$ and $b/a = k$, we obtain $V = 4/3\pi a^3 k$.

From the constant-volume condition $1/6\pi d^3 = 4/3\pi a^3 k$. Hence $d = 2ak^{1/3}$ and $a = {}^1/_2 \sqrt[3]{d^3/k}$.

We calculated the semiaxes and b for each droplet size. Ratio k was varied from 0.99 (almost corresponding to a sphere) to 0.1 with an interval of 0.05 and from 0.1 to 0.01 with an interval of 0.01.

Macrophotography of the deformed droplets made it possible to measure with sufficient accuracy (0.05 mm) the major and minor axes of the ellipse. A comparison of the measured semiaxis dimensions with the calculated data made it possible to determine the original diameter of the deformed droplet.

Apart from the dimensions of the deformed droplet, we simultaneously determined the following quantities: degree of deformation k, mean droplet velocity W_k on an interval of 10 mm, time τ taken by the droplet to traverse that interval, and the mean gas flow velocity.

For calculation and graphical purposes we took weighted means of many values obtained for different flow conditions in the Venturi tube, but standardized according to a single principle—identical degree of deformation of the droplet. In general, the deviation from the means of the limiting values for all the quantities measured did not exceed ±15%.

Figure 2 presents the results of an analysis for the experimental data in a plot of total drag coefficient (C_x) versus Reynolds number (R). In determining R we took the dimensions of the deformed droplet, i.e., R was a function not only of relative velocity U but also of the characteristic linear dimension of the droplet.

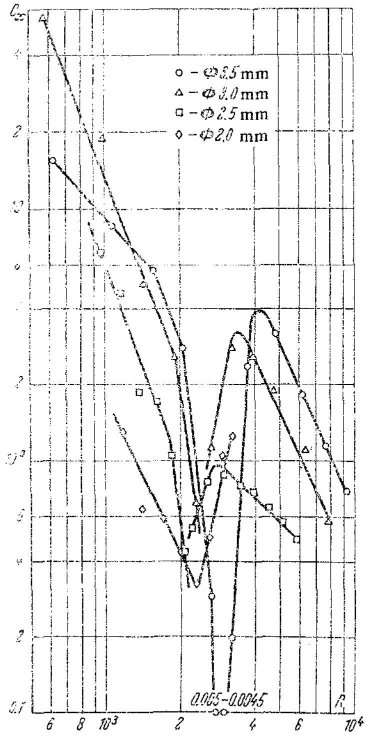

Fig. 2

The variation of C_X with R can be divided into two phases; in the first phase, corresponding to the stage of "pulsating" acceleration, there is a quite sharp fall in C_X as R increases. Thus, for d = 3.5 mm the coefficient fell to $C_X = 0.005-0.045$ at values of R from 2800 to 3000. A tendency for C_X to fall is also observed with droplets from 3.0 to 2.0 mm in diameter in a narrow range of R from 2000 to 2600; unfortunately, the minimum values could not be established.

The decrease in C_X can be explained as follows. Whereas a sphere is a blunt body for which pressure drag is important even at low speeds, for a liquid droplet during the first stage the action of the dynamic gas pressure continuously decreases owing to liquid circulation in the droplet.

The absence of slip at the phase interface leads to the gradual involvement of an ever greater quantity of the droplet liquid in the circulation, and, as the flow velocity increases in the convergent part of the Venturi tube, the circulation intensity becomes greater and greater. Owing to the circulation of the liquid, the actual increase in the relative velocity between the gas flow and the droplet is small; this also explains, in particular, the fact that for flow over a rigid sphere at R from 0.1 to 100 values of C_X are the same as for flow over a droplet at R from 500 to 2600.

In this stage flow over the droplet may also be unseparated; however, in principle the action of the gas flow on the droplet remains dynamic at these gas velocities; therefore, to calculate C_X we took a quadratic dependence of drag force on velocity.

As the diameter of the droplet decreases, owing to the increase in the Laplace pressure, which is inversely proportional to the droplet radius, C_X approaches ever more closely to the characteristic value for flow over a rigid sphere at the same R-values. Moreover, this is partially confirmed by the fall in C_X-values obrained for d = 2.0 mm (see Fig. 2).

The increase of absolute velocity for the droplet with time is shown in Fig. 3. The greater the droplet diameter, the sharper the transition between the two stages of deformation. For d = 3.5 mm, at a droplet velocity from 2.4 to 2.6 m/sec, and at a relative velocity 8,5 m/sec, there is a characteristic "plateau," whose presence is attributable to attainment of a maximum liquid circulation in the droplet, when the dynamic pressure effect is minimized. Thus, at very low values of the drag coefficient the resultant force acting on a droplet 3.5 mm in diameter is equal to $0.194 \cdot 10^{-3}$ N, i. e., almost equal to the force of

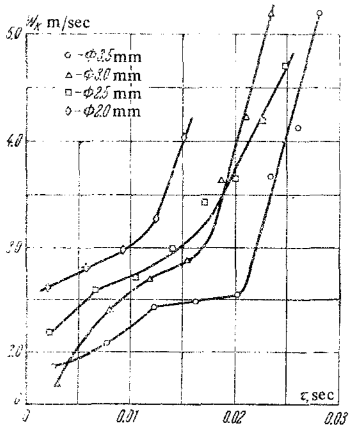

Fig. 3

gravity ($F_2 = 0.190 \cdot 10^{-3}$ N), while for droplets from 3.0 to 2.0 mm in diameter is two–four times greater than the force of gravity. Correspondingly, for droplets from 3.0 to 2.5 mm in diameter the "plateau" is less apparent, while at d = 2.0 mm it almost disappears.

In Fig. 4 end of the first stage and attainment of maximum circulation in the droplet correspond to a sudden increase in the degree of deformation (especially for droplets from 3.5 to 2.5 in diameter) for only a small change in relative velocity.

This suggests that an important role in relation to droplet shape is played, apart from dynamic pressure, by circulation in the droplet. Obviously, maximum circulation leads to the spherical shape losing its stability and going over into an ellipsoid of revolution. Partial proof of this is the strictly symmetrical shape of the ellipsoid with a degree of deformation k = 0.6, which was the same for all droplets on transition to the second stage of "regular" deformation. Action of the dynamic pressure alone would not ensure such strictly symmetrical deformation. As the droplet diameter decreases, greater relative velocity is required to attain the same degree of deformation and for droplets from 3.5 to 2.0 mm in diameter it is equal to 8.5–12 m/sec.

The change in the shape of the droplet at the beginning of the second stage of regular deformation leads to an increase in C_X in the same narrow range of Reynolds numbers R = 2200–3000 (see Fig. 2), which is, obviously, a consequence of sharp reduction of circulation inside the droplet and a transition to separated flow.

Further change in droplet shape from ellipsoidal (k = 0.6) to disk-shaped continues to k = 0.1–0.9, which corresponds to a disk with a

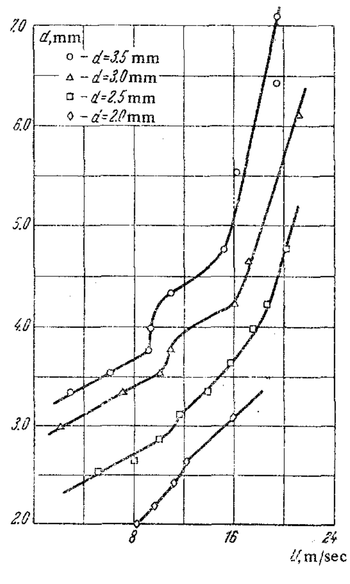

Fig. 4 .

diameter approximately equal to twice the diameter of the original droplet. Taylor [8] has called this the critical state, when even slight variations (pulsations) of the gas flow lead the disk to break up into smaller droplets. In this phase the drag coefficient falls steadily as Reynolds number increases in the range $R = 3 \cdot 10^3 - 1 \cdot 10^4$ (see Fig. 2) and only in the critical stage reaches values $C_x = 0.5$–0.8.

On average, however, the drag coefficient in this stage is two to six times greater than the coefficient for a rigid sphere ($C_x = 0.44$) and tends more toward the value of the dynamic coefficient for a flat disk ($C_x = 1.11 - 1.33$ at $R = 1 \cdot 10^3 - 3 \cdot 10^5$) [9].

Thus, experimental results lead to the following conclusions:

1. The liquid nature of the droplet and droplet deformation lead to important deviations of the total drag coefficient C_x from the value of a rigid spherical body at the same Reynolds numbers.

2. The value $C_x = 0.44$ (drag coefficient of a rigid sphere in the self-similar regime) used for calculating the velocity and breakup energy of liquid particles is clearly too low and does not follow from the actual interaction pattern. On average, the experimentally established value of C_x in the breakup stage for droplets from 3.5 to 3.0 mm in diameter is 4.5–6 times greater.

The dependence of drag coefficient on Reynolds number can be represented as $C_x = A/R^n$; here, for the "pulsating" stage of deformation in the range of R from 500 to 2500

$$\begin{matrix} d = 3.5\text{—}3.0 & A = 427 \cdot 10^8 & n = 3.15 \\ d = 2.5 & A = 108.5 \cdot 10^8 & n = 3.08 \end{matrix} \quad \langle n \rangle = 3.1\,;$$

for the "regular" stage of deformation in the range $R = 4 \cdot 10^3 - 1 \cdot 10^4$

$$\begin{matrix} d = 3.5 & A = 4.10^8 & n = 2.2 \\ d = 3.0 & A = 1.2 \cdot 10^8 & n = 2.13 \end{matrix} \quad \langle n \rangle = 2.15\,.$$

In the experiments a droplet with d = 2.0 mm could not be brought to breakup, and the data were obtained in a narrow range of Reynolds numbers. It is not possible to give analogous relations for this droplet. However, it should be noted that as the droplet diameter decreases there is a tendency toward equalization of the drag coefficient for the liquid droplet and that for rigid spherical body at the same Reynolds number.

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22 September 1966 Sverdlovsk